

# Chapter 8

## Embodied and Extended Numerical Cognition



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**Abstract** In this chapter we consider the theories of embodied cognition and extended mind with respect to the human ability to engage in numerical cognition. Such an enquiry requires first distinguishing between our innate number sense and the sort of numerical reasoning that is unique to humans. We provide anthropological and linguistic research to defend the thesis that places the body at the center of our development of numerical reasoning. We then draw on archaeological research to suggest a rough date for when ancient humans first were able to represent numerical information beyond the body and in enduring material artifacts. We conclude by briefly describing how these capacities for embodied and extended numerical cognition shaped our world.

**Keywords** Number words · Counting · Embodiment · Extended mind · Numerical cognition · Calendars

### 8.1 Introduction

What is the relationship of embodiment to our capacity to think numerically? It might seem, at first, that the capacity for numerical thought would be a paradigm case for the computational theory of mind. However, as we argue, numerical thought is embodied.

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Consideration of the relationship of embodiment to our capacity to think numerically requires first distinguishing quantal from numerical cognition. Quantal reasoning, QR, is present during infancy, shared with other species, and allows us to discriminate quantities up to three, automatically, without counting. By contrast, numerical reasoning, NR, the ability to discriminate quantities greater than three, requires us to have acquired number words. As we will defend, the genesis of number words stems from the special relationship humans have with our fingers and toes. In this way, NR is necessarily embodied. By contrast, QR is not.

The development of number words was the first important shift enabling the advent of NR; a second important shift took place when we developed the ability to represent quantities outside the body. We discuss the first evidence of numerical cognition as represented in our digits and in non-digital artifacts, and consider how the latter development of non-anatomical numerical representation shaped our capacities to think. We see the evidence we present here as providing support for the criticality of the body and external representations to NR.

We present this discussion in terms of philosophical theories of embodied and extended mind. The application of such theories will be broad. Concomitantly, we hope to steer clear of ongoing philosophical debates regarding the boundaries of the mind, and of associated debates regarding the usage of terms like ‘belief’, ‘consciousness’, and ‘cognitive’ (Chalmers and Clark 1998; Prinz 2008; Adams and Aizawa 2008).

### 8.1.1 *Numbers in Computational Theory of Mind*

Numerical reasoning has historically been understood to be a paradigm case of the sort of processing that can be done by computers (Rescorla 2017). Although scientists and programmers continue to develop more and more lifelike AI, mathematical problem-solving is one area where computers have long excelled. Although Siri might be bad at understanding some of your requests, no one doubts her ability to do math, or the fact that a graphing calculator can create a graph with more speed and accuracy than anyone can by hand. Computers have calculated sums that would be impossible without technology. For example, with computers working 24 h a day for 105 days we have recently been able to calculate 22,459,157,718,361 digits of pi (Revell 2017). More practical and equally impressive applications of the mathematical capacities of increasingly rapid silicon (and non-silicon) processors abound in more commonplace devices and programs—from spreadsheets to statistical analysis platforms, to the algorithms at the core of many common apps for social network analysis, direction-finding, and internet searches. Binary-based computational math pervades modernity.

Because of such facts about computers, theories of embodiment may not seem to have their most natural home with numbers. But a different question—and the one we will consider in this chapter—is whether *our* numerical capacity supports the computational theory of mind, or if NR is evidence of the theories of embodied and

extended mind. We will argue in the following sections that the body and artifacts outside the body played an essential causal role in the development of numbers, and continue to bear an important connection to how we conceive of and use numbers today.

### 8.1.2 Embodied Cognition

Before we continue with our discussion of numbers and forms of numerical cognition, some philosophical background is needed. The view of the mind as something distinct from the body was most famously presented in Descartes' *Meditations* in 1641. As highlighted in his contemporaneous letters with Princess Elizabeth of Bohemia, Descartes draws a fundamental and problematic distinction between the self as a thinking thing and the self as a body made of matter (Atherton 1994).

If there is nothing special about our selves as situated in or constituted by our bodies then computers could be an apt metaphor for the mind. This view had its heyday in the 1960s and 1970s but in the past 20 years there has been pushback against such theories, and many philosophers working on cognitive science and consciousness today have rejected the computer as a metaphor for the mind (Rescorla 2017; Churchland 2017; MacFarquar 2018). Many philosophers focus instead on what is special about ourselves as embodied beings, and, to further extend the realm of the self, on how the mind may extend from the body and to the tools we use (Chalmers and Clark 1998; Prinz 2008; Adams and Aizawa 2008; Menary 2015; MacFarquar 2018).

In a well-known philosophical thought experiment, we are asked to consider having our brains removed and put into a vat (Harman 1973). We are to imagine that all our sensory experience would be simulated by computer programs connected to our brains. According to proponents of the embodied cognition thesis, this thought experiment could never be fully realized, regardless of the advancements of science—because being me, and experiencing the world as I experience it, is inseparable from my experience in a physical body (Prinz 2008). According to the embodiment theorist, any agent's experience could never be accurately simulated by the brain alone, for that experience is inextricably tied to bodily states (Prinz 2008).

Some critics caution that the claims of the embodiment thesis may have been overblown, overhyped, or plain wrong (Prinz 2008; Adams and Aizawa 2008). Jesse Prinz writes that although embodied cognition is trendy, “the philosophical equivalent of a blockbuster”, “excitement is not always correlated with truth” (Prinz 2008). Prinz argues that the theory of embodied cognition does not “hold the basic key to explaining consciousness” and argues for the more minimal thesis that “certain aspects of consciousness may depend on systems involved in perceiving and controlling the body” (Prinz 2008, pp. 1–2). Although we make use of the embodied cognition thesis here, we do not weigh in on its precise relationship to consciousness. Our argument is consistent with Prinz's more minimal take on the theory, as

well as with a construal of the embodiment thesis that places consciousness at its center.

### 8.1.3 *Extended Mind*

The extended mind thesis expands the notion of the self beyond the mind and the body—as in the embodied mind thesis—and into the external world. As Dave Chalmers and Andy Clark argue in their canonical 1998 paper, our relationship to tools in the world, such as reminder notes, bears certain similarities to our relationship to our memory. They present us with the case of Otto, who has Alzheimer’s and relies on his notebook to remind him of things, such as that the Museum of Modern Art is on 53rd St. In other words, as Chalmers and Clark put it, “his notebook plays the role usually played by biological memory” (1998, p. 12). They argue that because of examples such as this, we ought to say that Otto has *beliefs* that are in his notebook, rather than in his memory.

Chalmers and Clark’s extended mind thesis highlights the question of whether belief is necessarily an internal mental phenomenon, or if a belief can be understood as an externally accessible, action-guiding proposition. If the former, then Otto does not have the belief that the MoMa is on 53rd St, if the latter, he does. Chalmers and Clark argue that even if we do not ordinarily use the word ‘belief’ in this way, we *ought* to, because such a notion of belief is more explanatorily useful (1998, p. 14).

In describing the extended mind thesis Fred Adams and Ken Aizawa put the theory as follows: “This is the view that when a student takes notes in class, the student literally commits information to memory. When someone uses pencil and paper to compute large sums, cognitive processes extend to the pencil and paper themselves” (Adams and Aizawa 2008, p. 79). Adams and Aizawa go on to argue that, contra the extended mind thesis as they see it, tool use is a case of “cognitive processes interacting with portions of the noncognitive environment”, rather than “a matter of cognitive processing throughout” (Adams and Aizawa 2008, p. 80). They recognize that there is a potential for such discussions to devolve into terminological debates about words such as ‘cognitive’. However, what they argue is that the cognitive should be understood as particular to the processes of the brain (Adams and Aizawa 2008).

Whether it is more explanatorily useful to speak of beliefs as things that can be in notebooks, as Chalmers and Clark argue, or as mental states that must be ‘in the cranium’, as Adams and Aizawa argue, remains to be seen, and we do not wish to weigh in on these ongoing debates. However, the broader point that Chalmers and Clark make is relevant to our discussion. There are certain external tools in the world that we use in a way that is similar in important respects to how we use memory. Through the creation of symbols that can create occurrent belief states we are able to offload certain tasks beyond the brain itself. At some point in our history this happened with numbers. In later sections we will propose when this transition occurred.

### 8.1.4 *Our Aims*

Before we proceed with presenting linguistic evidence that supports our view, let us say a word about the framing of this chapter. Our discussion here is presented in terms of philosophical theories on embodiment and the extended mind thesis. These are philosophical ideas that, although enjoying more prominence in the past 20 years, are not without their nuances and critics (see above and Prinz 2008; Adams and Aizawa 2008; Menary 2015; Rescorla 2017). In this discussion we characterize one range of viewpoints as the theory of embodiment and another range of viewpoints as the extended mind hypothesis; we attempt to stay out of some of the debates within the literature on embodiment and the extended mind. This is a necessary step to being able to say something about how these theories relate to the linguistic and archaeological evidence we present, without getting mired in the philosophical details.

The theories of embodiment and the extended mind are the framework on which we present and frame our ideas and findings about number words and counting. Some other theorists who have discussed numbers and theories of embodiment or extended mind have done so with the aim of defending some *particular* position within the debates on embodiment or the extended mind thesis—or some other position within this space such as ‘radical enactivism’ (Zahidi and Myin 2016) or ‘cognitive integration’ (Menary 2015)—with numerical cognition as their specific evidence in favor of their preferred view of the mind. This is *not* our aim. Our aim is not to defend any particular theory of the mind, but to sketch out a more general version of the embodied mind thesis and extended mind thesis and demonstrate how linguistic and archaeological evidence fits within this picture. When attempting to connect disparate threads across disciplines it is necessary to paint with a broad brush; this makes it possible to situate research from linguistics and archaeology within a simplified philosophical framework.

## 8.2 Cross-Cultural and Cross-Linguistic Evidence Highlights the Embodied Bases of *Numerical Cognition*

Now that we have presented the philosophical framing, we will now present empirical research that supports the thesis that humans’ acquisition of number words was embodied. We also offer evidence that this acquisition was essential to the advent of NR, which is unique to our species, in contrast to QR. In the subsequent section we will return to the extended mind thesis.

Recent work in the field of cognitive science, based in large measure on cross-linguistic and cross-cultural studies, has aimed to draw a distinction between two kinds of cognition: numerical cognition and ‘quantical’ cognition. Two similar efforts to elucidate this distinction were made in recent publications: Núñez (2017) and Everett (2017). While Everett (2017) draws a distinction between quantitative

(as opposed to ‘quantical’, a term coined by Núñez) and numerical thought, the relevant claims made in these works are remarkably similar. Both authors contend that there is a pressing and heretofore unnoticed need to dissociate those facets of quantitative cognition that are innate and generally inexact, henceforth *quantical* cognition, from the culturally and linguistically contingent *numerical* cognition that allows humans to count and, more broadly, to precisely distinguish all quantities. Núñez and Everett suggest, independently, that numerical and quantical cognition are inappropriately conflated in research on how humans think with and about quantities. They highlight clear drawbacks associated with this conflation, principally the muddying of the nature of humans’ native capacities for discriminating quantities and the associated muddying of the role that these native capacities have in generating more elaborate forms of numerical thought. It is worth briefly outlining and disentangling numerical and quantical thought, and the empirical bases on which Núñez (2017) and Everett (2017) rest their claims. We do so next, prior to outlining the ways in which embodied processes were (and are) essential to the development of uniquely human *numerical* thought.

Humans, like a variety of other species, appear to possess a native and abstract capacity for distinguishing quantities. Shortly after birth we are capable of comparing and discriminating the quantities of given sets of stimuli, if the ratio between the sets is sufficiently large. As one example of this abstract capacity, recent experiments with day-old infants demonstrated that babies are generally capable of distinguishing 18 colored dots from, say, 6 dots, even after confounding variables (such as absolute size of stimuli, stimuli movement, etc.) are controlled. In such a case, the ratio between the two sets of stimuli is pronounced at 3:1, facilitating discrimination. Yet prelinguistic infants are also capable of consistently discriminating sets of items if the ratio describing their discrepancy is as low as 2:1. With ratios lower than 2:1, infants struggle with differentiating the relevant quantities. This native capacity for discriminating sets, assuming the ratio describing their respective amounts is sufficiently large, is often referred to as the ‘approximate number sense’ (Dehaene 1997). The abstract nature of this apparently native sense, which is typically housed in the intraparietal sulcus judging from a host of cortical imaging studies (Everett 2017), is evident in the cross-modal nature of some of the sets of stimuli discriminated by infants. Gaze tasks suggest that infants recognize, approximately, the quantitative similarity of a given set of audio stimuli, like a series of beeps, and an equinumerous set of visual stimuli, like dots (Xu and Spelke 2000; Izard et al. 2009).

Humans are also natively equipped with the capacity for exactly discriminating quantities less than four. This is evident in work with prelinguistic children (Wynn 1992), but also with adults in numberless cultures (Spaepen et al. 2011; Everett and Madora 2012, *inter alia*). Some debate exists as to whether or not this exact discrimination is truly distinct from the approximate ‘number’ sense, or whether our ability to discriminate smaller quantities follows from, e.g., the fact that the ratio between small quantities like 1 and 2 is sufficiently large to allow for their consistent subitization (Piantadosi 2016).

Setting aside such concerns, the abilities to subitize small quantities and to approximately discriminate large quantities are often referred to, together, as humans' innate 'number' sense and as a key part of our 'numerical' cognition. It is generally agreed, and certainly not contested here, that these innate capacities buttress the edifice of other more exact forms of numerical cognition, including arithmetic and the like. What is contestable is whether our imprecise native abilities should be referred to as 'numerical'.

Furthermore, while it is generally agreed that humans' native capacities for discriminating quantities is critical to the scaffolding of more robust forms of numerical thought, intense debate persists as to how this scaffolding occurs ontogenetically, or as to how it occurred diachronically (Overmann 2015; Everett 2015). Much of the motivation for this debate also underscores a key reason it is problematic to refer to humans' native quantitative reasoning as 'numerical cognition': these native capacities are largely similar to the capacities observed in many other species. Species with quantity discrimination skills that are at least roughly similar to those observed in prelinguistic infants and numberless adults include a variety of other primates, as well as a host of phylogenetically distant vertebrates (Brannon and Park 2015; Agrillo 2015).

It is unclear whether the quantitative skills of some of these species are homologous or analogous features, but it is increasingly clear that some of these skills are pervasive in nature. This pervasiveness makes troublesome the common terminological choice, evident throughout the cross-disciplinary literature on this topic, to refer to native human quantitative capacities as 'numerical' or as evidence for a human 'number sense'. After all, there is a gross discrepancy between these pervasive abilities, shared by so many species, and the ability to, say, distinguish 6 from 7 items consistently—an ability only observed in human populations that have acquired words and other symbols for exact quantities. As Núñez elegantly notes:

The adjective 'numerical' in 'numerical cognition', however, is crucially over-inclusive: any cognition or behavior relating to quantity in babies, monkeys, rats, or fish—whether exact or inexact, symbolic or non-symbolic, operational or not—is labeled as being 'numerical'. This loose over-inclusiveness licenses stating—teleologically—that thousands of species, from fish to humans, by virtue of being able to discriminate quantities, de facto have 'number representations' as a result of biological evolution (Núñez 2017, p. 419).

The common and unfortunate terminological choice licenses implicit assessments of the nature of elaborate human *numerical* cognition, which is unique and only evident in those members of the species who are also members of numerate cultures. This sort of cognition clearly does not owe itself simply to basic neurobiology, given that it is not a cross-cultural universal, and is not simply a natural byproduct, nor even a straightforward cultural refinement, of our innate 'number sense' since that 'number sense' did not evolve for actually numerical purposes.

The relationship between truly numerical cognition and the coarse quantitative distinctions enabled by our neurobiology is hardly direct—as evidenced by the lack of precise numerical cognition in other species that seem to share many of our native abilities for quantitative reasoning. (Excepting certain members of other species that have been laboriously trained with number words and symbols, who also

share some of our numerical cognition but concomitantly highlight the essential role that numbers play in enabling truly numerical thought—see discussion in Everett (2017).) For such reasons, Núñez (2017) suggests that our relevant biologically endowed capacities be referred to as ‘quantal’, rather than ‘numerical’.

Numerical cognition, while relying on a phylogenetically primitive system for quantal cognition, is culturally and linguistically dependent. A fair amount of cross-cultural and developmental data now suggests that humans lack the capacity for consistently and exactly discriminating most quantities prior to their acquisition of numbers—words and other symbols for precise quantities (Everett 2017). From a diachronic perspective, the cultural acquisition of numbers is somewhat haphazard and contingent on a host of idiosyncratic factors including patterns of contact between diverse linguistic communities.

Nevertheless, at its core the process of the cultural acquisition of truly numerical concepts (rather than quantal ones) has an embodied etiology. While scholars have long recognized that number systems are often motivated by human anatomical characteristics, the extent of these physical motivations has still been underappreciated. Next we offer a brief overview of the cross-linguistic evidence that demonstrates the extent to which human anatomy motivates both the germination and florescence of linguistic systems of numbers, and thereby ultimately allows for truly numerical cognition across the bulk of the world’s cultures.

As background to this overview, though, consider how quantal information surfaces in the world’s languages: the vast majority of the world’s languages make grammatical, rather than lexical, distinctions between the quantities distinguishable with our innate capacity for quantal reasoning. Most notably, the bulk of the world’s languages distinguish grammatically between one entity and more than one entity via a singular vs. plural distinction. This basic distinction surfaces in nominal plurality/singularity, verb-subject agreement patterns, and sundry other grammatical phenomena. In rarer cases grammars also distinguish between singular and dual categories of entities, or, rarer still, singular, dual, and trial. Other potential categories, principally the paucal and plural categories, are imprecise. In other words, what the world’s grammars discriminate correlates neatly with what native quantal reasoning allows speakers to discriminate: 1, 2, 3, and other larger quantities. This seems unlikely to be a coincidence, a point advanced recently in Everett (2017) and Franzon et al. (2019). (Though it should be acknowledged that most languages do not utilize a grammatical dual or trial, meaning that there is hardly an exact correspondence between quantal reasoning and grammar across all the world’s languages.)

In contrast, truly numerical concepts, related to higher quantities that are not precisely discriminated with our native hardware, are not encoded morphologically or syntactically, but lexically, in the words themselves. Precise words for higher quantities exist in nearly all, but critically not some, of the world’s languages. Tellingly, these words for quantities have clear anatomical bases in the vast majority of the world’s languages. This fact is evidenced in myriad ways, including the typological commonality of number words with decimal bases. While the commonality of decimal bases has been acknowledged for many years, though, its extent is still



perhaps underacknowledged. So, it is worth considering what the cross-linguistic data say about the extent to which the human hands facilitate the creation of number words.

In many languages the words for five or ten are etymologically related to the word for hand. Consider some examples from two Amazonian languages, Jarawara and Karitiâna, on which one of us has done field research. In (1) and (2) we see the words for five and ten in Jarawara:

- (1) yehe ohari  
hand one  
“five”, literally “one hand”
- (2) yehe ka-fama  
hand with-two  
“ten”, literally, “with  
two hands”

In (3) the word for five in Karitiâna reveals the same manual source:

- (3) yj-pyt  
our-hand  
“five”, literally  
“our hand”

This manual basis is also evident for larger Karitiâna numbers like eleven, as we seen in (4):

- (4) myhint      yj-py      ota                  oot  
one          our-hand      another take  
“eleven”, literally “take one and our  
other hand”

Many of the world’s languages have such transparently manual (and ultimately digital) origins. The word for five and/or ten is the lexical base on which higher numbers are constructed in languages like Jarawara and Karitiana, with numbers like ‘six’ and ‘eleven’ taking the form of ‘five plus one’, ‘ten plus a finger’, and so on.

The word for five serves to kickstart, diachronically, the growth of number-term systems in the world’s languages. This is evident in a recent survey of Australian languages, which are not nearly as anumeric as some presume (Bower and Zentz 2012). Worldwide, anatomically motivated words for ‘five’ and ‘ten’ seem critical to the intra-linguistic growth of number systems, and serve as the base for most number words. These words then become critical to the transmission of precise numerical concepts cross-culturally and cross-generationally.

Surprisingly to some, humans do not seem to grasp the associated numerical concepts prior to learning the relevant number words. Framed differently: the

manual basis of number systems is not simply the result of convenient labels of pre-existing numerical concepts acquired through our ‘number sense’, the manual basis is what allows for the acquisition of numerical concepts. One wonders, then, how exactly the fingers/hands enable(d) some people to transcend quantal reasoning and create precise numbers for higher quantities, entering the world of the truly numerical. We address this issue below.

While languages like Karitiâna and Jarawara have number words with transparently digital origins, numbers are manually-sourced in most cultures, though the manual sources are evident more subtly. In most languages, numbers are decimally based. This is true in English, as evidenced in words like ‘twenty-one’ and ‘thirty-one’, wherein we begin counting at ‘one’ again with the addition of each ten units—words that are derived historically from a multiplicative and additive strategy that is now somewhat opaque (e.g., ‘twenty-one’ derives from ‘two-ten-one’).

Nevertheless the manual basis of English is somewhat obscured by the lack of a discernible etymological relationship between five and/or ten and ‘hand’ or ‘hands’, and by the fact that some number words like eleven and twelve are not as transparently decimal in their structure as their corresponding number words are in many languages (like Mandarin). Furthermore, English-speaking cultures generally use other bases, like the sexagesimal and duodecimal, for time-telling and navigation, further obscuring the manual historical underpinnings of all our numbers. We do, however, use the English word ‘digit’ to refer both to numerals and to our phalanges.

The manual basis of English numbers stretches back millennia, prior to the advent of written numerals in Mesopotamia and other regions. Both historically and ontogenetically, verbal numbers predate other kinds of numbers. (The written numerals we use today, with their own decimal basis of Indic origins, are a particularly recent innovation.) Work in historical linguistics has demonstrated that Proto-Indo-European used a decimal system, and this ancestral tongue was likely spoken over 6000 years ago. Most of the world’s people speak a language that has a decimal base. Both Proto-Indo-European and Proto-Sino-Tibetan had decimal bases, and speakers of Indo-European and Sino-Tibetan languages, which have inherited these decimal bases from millennia ago, represent over half of the world’s population. The ancestral tongues from which the other most pervasive language families descended, including Proto-Afro-Asiatic, Proto-Austronesian, and Proto-Niger-Congo, also had decimal number systems (Everett 2017).

Yet the current pervasiveness of decimal systems is not simply the result of the success of a few language families over the last few thousand years or so. Comrie (2013) conducted a worldwide survey of the kinds of number systems evident in the world’s languages. His sample consists of 196 languages. While there are about 7000 mutually unintelligible languages in the world today, Comrie’s sample represents all major families and regions, and is a reasonable indicator of patterns in all languages.

Of 196 languages, Comrie observes that 20 have limited number systems. These include ‘one-two-many’ systems that are found primarily in Australia in Amazonia. Of the remaining 176 languages that have robust systems of number, 125 (71%) are decimally based for numbers beyond ten. (Often smaller numbers are quinary based,

even in languages in which higher numbers are decimally based.) Twenty-two of the languages (12.5%) have hybrid decimal/vigesimal systems, while 20 (11.4%) have pure vigesimal systems. Framed slightly differently, about 95% of the languages with robust number systems have numbers that are digitally based—oriented according to the numbers of fingers and/or toes on the human body. Furthermore, of the remaining nine languages, five of these have numbers that are based on an ‘extended body-part system’. According to Comrie’s survey, then, less than 3% of languages with robust number systems structure their numbers around something other than features of the human body.

With exceedingly few exceptions, languages with number words greater than ‘five’ create numbers via the body. It should be noted that the ‘exceptions’, while not cases of limited number in the strictest sense, also do not include number systems that are open-ended, with limitless numerical referents. For example, the senary (base-6) systems of some languages in New Guinea, which owe themselves at least partially to the manner in which yams are stored in groups of six, are not generally used in elaborate counting and arithmetic. In short, the body has clearly been critical, more critical than is often realized, to the development of number words.

The claim that the body is critical to the historical development of number words, across the vast majority of the world’s cultures, suggests that the body is critical to the entrance of cultures into the world of numerical, rather than quantal, concepts. A surfeit of experimental data has demonstrated that number words are essential to children acquiring basic numerical concepts like the one-to-one correspondence between large sets. Work with anumeric adults has converged on the same conclusion (Spaepen et al. 2011; Everett and Madora 2012).

Rather than simply serving as labels for concepts that humans are natively pre-disposed to acquire during development, number words work as placeholders for concepts that children realize they must acquire. Learning numbers is not a matter of labeling concepts but of ‘concepting labels’ (Everett 2017; Carey 2009). Relatively early on during language acquisition, kids learn that number words come in a sequence, but do not appreciate that this sequence represents a growing magnitude. At a critical stage they learn the *successor principle*, realizing that each number word represents ‘one more’ than the quantity that precedes it. Native quantal reasoning appears to facilitate the acquisition of this principle, since kids can naturally recognize that two is greater than one, and that three is greater than two. Nevertheless, debate remains as to the role that native quantal reasoning plays in the acquisition of numerical concepts like ‘five’. This is true, at least in part, because other species possess quantal reasoning yet do not enter the world of the numerical unless they are trained with numbers of human origins. Interestingly, most kids also rely on their hands and finger-counting as they learn the successor principle. So the structure of the human body remains critical to the ontogenetic acquisition of number words like ‘five’ that were once created by innovators who also benefited from the structure of the human hands during the relevant innovation.

Ultimately, then, precise numerical concepts owe themselves either directly or indirectly to humans’ engagement with their fingers and, to a lesser extent, their

toes. Why are our bodies so critical to transcending the quantal? Why have humans been able to rely on their manual digits to transcend the quantal while other primates have not? The facile answer is that we are the only linguistic species, capable of naming quantities via patterns in our hands. Or perhaps—to make the point in a more sophisticated way—the mechanism or ‘module’ that underpins our acquisition of grammar also underpins our capacity to recognize cardinality, a requirement for counting (Hauser et al. 2002; De Cruz 2008). Yet language is not a sufficient criterion for the acquisition of numerical concepts, as evidenced by linguistic yet anumeric people (Everett 2015).

And if acquiring numbers, even numbers as small as ‘five’, is not simply a matter of acquiring labels for things that the world carves up (since most anumeric people cannot consistently discriminate 5 from 6 of the same item<sup>1</sup>), how exactly do the hands and fingers really enable numerical concepts? As noted above, a key numerical concept is the appreciation of one-to-one correspondence between sets larger than four. Critically, humans and other species without number words or other number symbols are not able to consistently appreciate one-to-one correspondence for such sets. Human hands may facilitate the innovation of numbers because they are symmetrical and expose us continually to visual and tactile (or even proprioceptive) one-to-one correspondence for two naturally occurring sets of five items. Five is—critically—larger than the precise quantities we can distinguish with quantal reasoning.

While other primates have quantal reasoning and even symmetrical hands, no other species has these characteristics *and* bipedalism, which at least partially explains humans’ unparalleled manual focus. This manual fixation apparently allows for the occasional and haphazard innovation of number words like ‘five’/‘hand’ by number inventors who concretize the otherwise ephemeral realization that the quantity of fingers on one hand is equal to the quantity of fingers on another, or perhaps the realization that the quantity of fingers on one hand is equal to a number of small valuable items organized on palm of the hand, alongside those fingers. Of course there may be other characteristics of our species, including neurophysiological ones, that also draw humans into the world of the numerical. But the characteristics of our hands, and our continuous engagement with our non-locomotive appendages, were critical to the genesis of NC judging from the extant cross-linguistic, cross-cultural, and cross-species data. Our capacity for numerical thought and use of number words is causally tied to our existence as embodied beings with ten fingers and toes.

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<sup>1</sup>This does not mean, of course, that a parent would not, say, be able to tell that one of her six children is missing from a lineup. Similarly, if we saw a map of the United States with (say) Florida missing, we would surely notice. This does not require us to count to 49 or suggest we are using numerical concepts. In neither of these cases does the recognition that something is missing require numerical reasoning but visual recognition that something is off from the norm. Visual recognition only goes so far, however, and very large families in our society may resort to counting to make sure the entire family is present on certain occasions. Thanks to Sean Allen-Hermanson for posing the question with the first example.

## 8.3 Archaeological Evidence

### 8.3.1 *The Extended Mind and Archaeological Evidence of Numerical Representation*

A significant further development in the capacity for numerical thought occurred when ancient humans moved beyond the ability to represent numbers linguistically, and were able to represent them in lasting symbols. It was this step that allowed our relationship with numbers to move beyond embodied cognition and into the territory of the extended mind thesis.

Recall from the introductory sections that the extended mind thesis, as proposed by Chalmers and Clark in their 1998 paper, concerns the ways our relationship to external tools can mirror our relationship to non-occurrent, but accessible, mental states. Chalmers and Clark ask us to consider Otto, the man with Alzheimer's, who uses a notebook to remind him that the MoMA is on 53rd St. A parallel is drawn between the sentence 'The MoMA is on 53rd Street' as written in the notebook and an internal mental state that Otto could have had with this same content.

As noted earlier, we hope to use this philosophical machinery without weighing in on its most controversial potential implications. That is, we do not wish to weigh in on the debate surrounding whether or not Otto's notebook is an extension of his mind and if the contents therein constitute Otto's mental states. However, drawing on the more modest construal, what the extended mind thesis does is highlight the ways certain external representations can change cognition and make accessible mental states that would be impossible without the use of tools. What we will consider now is the ways external representations of numbers changed our capacities to think numerically, and what archaeological remains constitutes evidence of numerical representation.

### 8.3.2 *Numerical Representations*

Archaeological evidence is such that it is necessarily a record of *externalized* mental states, that led to behaviors which modified the world in such a way that an enduring material record was created. Empirical facts about decay sharply reduce the quality of this evidence. Despite these limitations—which should be kept in mind throughout this discussion—a number of archaeologists have attempted to find the point at which humans first developed number tools that extended beyond the body, a change that extended our capacity for numerical cognition.

Today mathematicians do calculations by manipulating the Arabic numerals that we have adopted around the world to represent numbers, or by having computers do this for them. One reason that the Arabic numeral system overtook the system of Roman numerals that was previously used in the West is because mathematical calculations are extremely difficult to do with Roman numerals. Multiplication of

the quantities twenty-seven by twenty-seven is much easier to do when it is depicted as  $27 \times 27$  rather than as XXVII by XXVII.

In the previous sections we detailed a number of the ways that number words in societies across the globe bear evidence of the connection to the body at their genesis. This connection to the body is also seen with the numerical representation of Roman numerals. Unlike the Arabic numerals which provide an edge in calculations, the Roman numerals can be thought of as bearing a resemblance to what they represent. A single I in Roman numerals resembles one finger; a V resembles a hand, or five fingers, and X resembles two hands together, or ten fingers total. With Roman numerals we again see the primacy of the body, and this time not just in the spoken word itself but in the way that number is externalized and represented on material objects.

### 8.3.3 6000-Year-Old Clay Tokens and the Mind

Our current practice of depicting quantities with the Arabic numeral system is a result of a process of humankind developing numerical representations and keeping those that best suit our purposes. It is far from given that such representations would resemble Arabic or Roman numerals, and archaeologists must develop some account of what sorts of items from the archaeological record were used to represent quantities. Lambros Malafouris, in his 2013 book *How Things Shape the Mind*, engages with some of the same philosophical topics as we have here, including the extended mind thesis as applied specifically to numerical cognition, and argues that clay tokens found in 4000 BCE are the first archaeological evidence of numerical thought (Malafouris 2013, p. 113).

Rather than steering clear of the most controversial philosophical implications of the extended mind theory, as we attempt to do here, Malafouris embraces a radical interpretation of the extended cognition view wholeheartedly in his work. As Colin Renfrew writes in his laudatory introduction, Malafouris examines how “the human mental capacities that have their primary location in the brain (within the skull) are not separable in any serious consideration from their expression in action” (p. ix) and argues that “the mind is to be understood as embodied, indeed as extended beyond the body” (p. xi). In presenting his account, Malafouris makes the point in an even stronger way, writing that in the “gray zone of material engagement”, “brains, bodies, and things conflate, mutually catalyzing and constituting one another” (p. 5). The claim that brains, bodies, and things conflate is stronger than the idea proposed by philosophers who are proponents of the extended mind thesis.

The claim Malafouris makes in this discussion is to parse, and on at least one interpretation seems clearly false and open to a number of obvious objections. Does Malafouris mean to claim that bodies, brains, and things *literally* conflate? Clearly it is not the case because the 6000 year old clay tokens he discusses are still here and can be held in the hand of an archaeologist, while the bodies of whoever it was that made and used those clay tokens have been reduced to bones or less, and the brains

have long since decayed to nothing. Bodies, brains, and things persist over different timescales. It is this very fact that makes it worthwhile to take some thought, such as ‘MoMA is on 53rd Street’ and to put it down in a notebook. It is one thing to say that the notebook should count as a part of the mind and quite another to say that bodies, brains, and things conflate.

Malafouris does not attempt to get around such objections by clarifying precisely what he means by the claim that “brains, bodies, and things conflate”. He writes, “too much clarity and too great an emphasis on definitions could be misleading in a context where transgressing the common wisdom about minds and things is often a precondition for success” (p. 9). His argument certainly does not suffer from an overabundance of clarity. Malafouris says he is transgressing the common wisdom about minds and things but does not tell us how. On its face this makes his position seem implausible. Without clarifying what else he could have meant it remains so.

Perhaps in an attempt to be as charitable as possible to Malafouris we could characterize his view as close to the Chalmers and Clark position and take it to be that minds (not brains) are sometimes outside the skull, and even outside the body. Such a construal might make his view appear to be a straightforward application of the extended mind thesis to the archaeological record.

However, Malafouris himself makes it clear that this is not what his view amounts to. Instead, he writes that his conclusions go beyond what the philosophers he draws on were willing to commit to. On this point he writes:

Most philosophical treatments remain epistemically agnostic about material culture’s properties and about its active role in human life and evolution. Even embodied cognitive science (Anderson 2003; Wheeler 2005; Chemero 2009; Clark 1997, 2008), which explicitly recognizes the intrinsic relationship between brain/body and environment, often seems oblivious to the phenomenal properties of the material medium that envelops and shapes our lives. Although the material world is recognized as a ‘causal influence’ rather than a ‘mere stimulus’, it is rarely seen as playing a ‘constitutive’ role (Malafouris 2013, p. 10).

We see here Malafouris commits himself further to the position that the mind is constituted by things.<sup>2</sup> He argues in the quote above that philosophers’ failure to reach this conclusion is a result of their being ‘oblivious’ to materiality. Malafouris then expresses disappointment that the philosophers who developed the theories he draws on have not been led to his conclusion, writing, “at the present stage of research, philosophy of mind remains skeptical and undecided about entering the treacherous territory of the extended mind proper” (pp. 10–11). It is this purported failing of philosophy to take material culture seriously that Malafouris aims to rectify in his book.

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<sup>2</sup>It is not clear if Malafouris mistakenly believes that Chalmers and Clark (1998) hold the view the Otto’s notebook plays a ‘mere’ causal role rather than a constitutive one, or if he believes this paper (which, as of publication, has over 5000 citations) is one of the ‘rare’ exceptions to philosophers allegedly overlooking this possibility.

### 8.3.4 *Malafouris on Numbers in the Archaeological Record*

What does it look like when Malafouris's 'extended mind proper', as he sees it, is applied to the archaeological record? A fruitful place to look is his discussion of numbers, which, as we noted above, leads us to consideration of which artifacts in the archaeological record are evidence of numerical cognition. Our position is that the capacity to represent numbers in external tools that could then create occurrent mental states is an advancement that changed our capacity for different types of numerical cognition.

Malafouris adopts a more radical position with respect to number symbols. He writes:

...meaningful engagement of material signs is the precondition for the emergence of symbolism. These physical relations and interactions between the body and cultural artifacts should not be taken as mere 'indications' of 'internal' and invisible mental processes; they should, rather, be taken as an important form of thinking (Malafouris 2013, p. 105)

A few things are remarkable about the claims made here. First, this suggestion that material signs are a precondition for the emergence of symbolism calls into question what it was that motivated the creation of the material sign in the first place. If there was not first symbolism in the form of a mental representation, why would the material sign have been created? Malafouris frames his discussion in terms of the mental capacities we detailed in the previous section, and asks "Could *Homo sapiens* alone—that is, in the absence of external material support—have ever have [sic] moved beyond approximation" (p. 106)? By approximation, he means the sort of number approximation that we have called 'quantal'.

Most researchers believe that number words are what enable humans to move beyond approximation to exact number sense (p. 109), as we have defended here. Malafouris adopts a different position: that number words are not necessary to have number concepts (p. 110). On this point, he cites work by Daniel Everett with the Piraha and writes: "Another interesting possibility is that it isn't the lack of number names but the lack of a 'counting routine' or a 'technology for counting' that keeps the Piraha from developing exact numerical thinking" (p. 110). He asks us to consider how "humans conceive or grasp the quantity of 10 when no linguistic quantifier, and no symbol to express it, is yet available" (p. 110). Malafouris concludes this discussion with the statement that language "is not sufficient" to account for humans' development of the concept of number (p. 111).

Is it true that some *Homo sapiens* do not have a 'technology for counting'? If what we have defended above is correct, then, contra Malafouris, fingers represent a 'technology for counting' that is possessed by all humans. We need only look to the hands for a tool to represent the quantity of ten. As we have argued, the evidence that the hands have a longstanding connection to the quantity of 10 is found cross-linguistically. We do not need to look beyond the bounds of our bodies to understand how we utilized fingers—something visible, something symmetrical, something we can manipulate at will, something outside the brain itself, something with proprioceptive qualities, something all humans have—to develop number



words up to 5, 10, 20, and number systems that are grounded in these quantities. Indeed it is difficult to conceive of better ‘material support’ for such a task.

However, Malafouris proposes instead that numerical thinking arises with the creation of forms of external material support, such as clay tokens. He writes: “the emergence of symbolic numerical thinking, in the particular context I am discussing, begins with the invention of the clay-token system” (p. 113). In other words, this is an argument that we had number tools before we had number concepts. Malafouris makes this explicitly clear when he writes that “at this early stage of concrete counting the concept of number had not yet emerged” (p. 114). According to Malafouris, we got numbers after we began to represent numbers in the enduring material record. In the context he considers, around 4000 BCE. This conclusion goes against what we have defended here and, as Malafouris notes, against the received view. The idea that *Homo sapiens* had concrete counting, as with the clay tokens, before we had the concept of number strains credulity. The burden lies with Malafouris to explain how something so implausible, and that goes against the established view, could be true.

### 8.3.5 *Hands as Technology for Counting*

When seeking to find evidence of a ‘technology for counting’ in the archaeological record, we need not look to clay tokens from 4000 BCE but to the fingers of skeletal remains. Of course, the presence of this ‘technology’ is not evidence of counting or number concepts. Non-human primates also have ten fingers and do not have number concepts. But we are all endowed with this ‘permanent tool’ that has numerous advantages as a means of counting. Among other advantages, research has also shown that the physical manipulation of fingers aids in number acquisition in children, and being able to use the body in mathematical tasks improves performance (Nathan 2014). The body does serve as an always-accessible technology, but beyond this, our proprioceptive relationship with our hands means that there are additional advantages beyond merely the body as ‘tool’.

At the same time, there are certain limitations to using the fingers and toes as your technology for counting. Numbers counted on the fingers can only go up so high, and numbers cannot be ‘held’ and accessed later beyond when the counting is performed. The capacity for storing or recording numbers using only the hands is limited. There are advantages to creating additional ‘technology for counting’ beyond what we are naturally endowed with. An ideal tool could be created or modified when some mental state is active, ignored, and then returned to again to activate such a mental state. A tool that, for example, tells me that it has been 5 days since the last full moon would be helpful if I am in a culture that needs to track the tides. As with Otto’s notebook that reminds him the MoMA is on 53rd Street, such a tool could help to activate mental states that would be too much of a drain to hold available in memory.

### 8.3.6 *Artificial Memory Systems as Technology for Counting*

In papers spanning 20 years, archaeologist Francesco d’Errico has proposed that we understand certain artifacts to be what he calls ‘artificial memory systems’ or AMSs, objects “conceived and produced to store, process and/or transmit numerical information” (d’Errico 1998; d’Errico et al. 2003; d’Errico et al. 2018). Such artifacts fit the bill for what we have been describing as the role of tools in the extended mind thesis. In presenting such artifacts d’Errico writes, “A fundamental turning point in the evolution of human cognitive abilities and cultural transmission was when humans were first able to store concepts with the aid of material symbols and to anchor or even locate memory outside the individual brain” (d’Errico et al. 2003, p. 31). Notice that contra Malafouris, d’Errico talks of storing concepts in material symbols, not of the material symbols constituting mental states.

An example of what d’Errico understands as an AMS is the notched bone first discussed by Alexander Marshack. Marshack argued that such markings track lunar phases (Marshack 1991; Dehaene 1997, pp. 95–96; d’Errico et al. 2003, p. 32). In his book *The Roots of Civilization: The Cognitive Beginnings of Man’s First Art, Symbol and Notation* Marshack develops his hypothesis with respect to an Ishango bone from around 25,000 kya (Marshack 1991, p. 32). This bone has 59 notches and Marshack proposes that this is a two-month calendar (Marshack 1991, p. 30). Marshack himself notes that he has not established this hypothesis in any definitive way, writing “This first crude test of counts, worked out from the photographs and drawings of the Ishango bone, presents us with the *possibility*, then, that we may have a lunar phrasing and notation. It gives us no certainty, one way or another, but also it does not eliminate the lunar possibility” (Marshack 1991, p. 31). If we follow Marshack and d’Errico in their hypothesis that these notches were used to “store, process and/or transmit numerical information” in a way that allowed hominins to “locate memory outside the individual brain” that still leaves a good deal of room for interpretation about the specific numerical information contained therein. As d’Errico notes, “archaeologists have proposed a number of hypotheses to explain these markings. They have been interpreted as *marques de chasse* (marks recording the number of prey killed), devices to keep track of songs, or the number of people attending a ceremony, or other notational/calculation systems” (d’Errico et al. 2003, p. 32). Marshack’s proposal is perhaps the most well-known, but his results are far from definitive, as he himself saw.

In recently published work on a notched hyena femur from approximately 72–60,000 kya, found in Les Pradelles, France, d’Errico argues that the notches are number symbols, but stops short of hypothesizing precisely what these numbers tracked. d’Errico argues that this artifact is the farthest back in history that we have evidence of AMSs (d’Errico et al. 2018). Because *Homo sapiens* were not yet in Europe at this time, this means that these incisions were made by Neanderthals. d’Errico uses a number of techniques, including microscopic and morphometric study, as well as experimental reconstruction to support his hypothesis. If number notches are evidence of the existence of number words, as has been proposed

(Dehaene 1997, p. 95), then this means that Neanderthals had number words. They had the same ten fingers and toes as we did and perhaps their body played a similar role in the development of those number words as they did in ours. We also cannot rule out the possibility that the origin of number concepts lies with a shared common ancestor,<sup>3</sup> perhaps *Homo heidelbergensis*, or another hominin, if the divergence between the *Homo sapiens* and *Homo neanderthalensis* is to be found farther back in history (Gomez-Robles 2019).

Whether or not we would want to say that the tally marks created by *Homo sapiens* and Neanderthals represent an extension of the human mind would require weighing in on details of the extended mind thesis that we have attempted to stay agnostic to. At some point this specific question does become a mere terminological debate rather than a metaphysical one. However, to make the point in the most neutral terms, what such bones with tally marks clearly do represent is a new tool that allows us to call to mind information in a way that we were previously unable to. Adopting the term, AMS, or artificial memory system, allows us to identify artifacts that capture the spirit of the extended mind thesis, without committing ourselves to—or even going beyond—its most controversial construals, as Malafouris does. Such AMSs were technology outside of the body itself—external, enduring technology that was built on number words that were initially developed using our hands as the first ‘technology for counting’.

With recognition of the complex relationships between numbers and what goes on in the brain, what goes on in the body, and what goes on in the world comes a recognition of the different selection pressures that act on both. Regardless of where one stakes out territory for ‘the mind’, ‘consciousness’, ‘cognitive’, and so on (Chalmers and Clark 1998; Adams and Aizawa 2008; Prinz 2008; Malafouris 2013) it is clear that different processes lead to change in things that are bodily and things that are not. This is especially important to note when considering the developmental story, as we do here. Natural selection led to us having ten fingers and ten toes, as do many other mammals. The cognitive ability to develop number words using our fingers and toes was a later developmental step for our species, that perhaps, as d’Errico’s research seems to show, was shared with Neanderthals.

Language itself is culturally transmitted, as are tools. This means that language and tools can develop at a faster rate than we can change genetically (Tomasello 1999). As d’Errico notes in the conclusion to his 2018 paper:

...the invention of number symbols appeared very recently and has required no biological change. Our brain has not undergone specific adaptations in order to be able to use number symbols. This suggests that it is quite possible, and this is what we would argue, that these cultural exaptations have not required concomitant significant inheritable biological changes (d’Errico et al. 2018, p. 8)

d’Errico’s conclusion highlights the benefits of being able to recognize the different selection pressures and thus different timescale for change that are at play with genetic factors in the brain and body versus cultural factors in tools.

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<sup>3</sup>Thanks to Anton Killin for raising this point.

## 8.4 Summary and Future Research Questions

Our hands are both something we have a proprioceptive relationship with, and something that we can perceive visually. Because the prehistorical notches made on bone left a physical indentation that could be felt and tracked with the fingers (see the images in d'Errico 2018), these tools may have been perceived through touch as well as visually. When we say that fingers and toes have played a special role in the invention of numbers, we have not specified if this is a claim that there is something about the proprioception of the body that is essential, or if hands are simply playing a role as a 'walking abacus' that we perceive visually and all happen to be endowed with. If it is the former this says something more about the *embodied* nature of this relationship.

This is, at least in part, an empirical question that could be studied. One potential place to look would be to number word acquisition in the blind. With words in general, language acquisition in blind children occurs at the same rate as in sighted children (Gleitman and Newport 1995). Assuming this includes number words as well, this suggests that it is the proprioception of our fingers, and not the visual perception of them that is essential to gaining number concepts ontogenetically, at least for children in numerate cultures. Whether proprioception is critical to the introduction of numerical concepts in a culture, and whether it was essential to the invention of number words, is another matter entirely. We suspect that it was at least beneficial, given the tactile symmetry of the fingers that seems to facilitate the appreciation of one-to-one correspondence for quantities greater than 3. Yet it is also worth noting that vision alone is sufficient to allow for the transmission of numerical concepts. After all, children with amelia, and digit-lacking individuals more generally, can also learn numbers. But, again, this is for individuals in cultures that already have number words; it is another matter whether or not number words could come about in a society where everyone had amelia or was digit-lacking.

Our discussion of numerical cognition is focused on number words and counting. In this scope of focus it differs from some of the recent literature on these topics, which focuses instead on the computational or number manipulation side of numerical cognition (Menary 2015). Numerical computation requires number words and counting to be in place prior to the development of these capacities and would follow concomitantly (Zahidi and Myin 2016; Flegg 2002). Richard Menary focuses on the ways symbols are manipulated in a proceduralized way, which, as he notes, is a recent phenomenon (Menary 2015, pp. 11–14). Numerical computation is a rarefied practice in the course of human history and a discussion of this capacity must be grounded in the fairly recent history of the Greek, Chinese, Arab, and Maya worlds (Renfrew and Bahn 2012; Ansary 2009). Perhaps it is best to describe such a focus as mathematics, where our focus here has been its precursor numbers and counting, and on the cross-linguistic and cross-cultural data that supports this more general story.

And lastly, our focus was in ways narrower than those that consider calendrical systems, construed broadly, as their focus. In the previous section, we considered

calendars as tools of externalizing number systems, but not all forms of calendars are necessarily numerical. If we understand calendars as an external feature that “helps to recognize and record temporal events” (De Smedt and De Cruz 2011, p. 66) then features of the natural world, would be included, and do not require numerosity to play this role. Such natural features would include the phases of the moon, the flowering of certain plants, the location of constellations, and features as basic as the changing of the seasons, such as the changing of the leaves or the first frost. Johan De Smedt and Helen De Cruz (2011) have argued that Palaeolithic rock art that depicts animals with identifiably seasonal fur patterns or behavior is a sort of calendar because these depictions played the role of “storing ecologically relevant information about the seasons” (De Smedt and De Cruz 2011, p. 70). Ideally one would want more evidence that these depictions in fact played this role, but nonetheless this discussion of calendars does not make use of numbers, and thus is different from our focus here.

To put it mildly, the expansion of numerical thought has had pervasive effects on the human experience. The effects of this particular sort of embodied thought have been radical and transformative, and are obviously wide-ranging. Consider for a moment some of the cultural and material practices that are associated with or a direct result of the availability of numerical cognition—practices that would not be possible were we to rely only on quantical cognition as humans have done for the bulk of their existence, and as some still do. The discrimination of time in discrete units that can be enumerated, and the general division of time, is the result of numerical cognition. The manner in which most of us demarcate the progression of time, governed as it is by an esoteric and vestigial Mesopotamian base-60 mathematical system, is possible only with numerical cognition and with particular number bases. Seconds, minutes, and hours are some of many non-material numerical constructs that help to govern our experience. More fundamentally, the tracking of days and lunar cycles, natural as opposed to cultural phenomena, requires numerical cognition. It is unlikely a coincidence that societies with very infrequent references to time and temporal progression, like the Tupi-Kawahib or Pirahã of Amazonia, are societies with few if any numbers.

At the material level, examples of the pervasive influence of numerical cognition also abound. We suspect that, as you read this, few if any of the human-made items in your surroundings—from smooth walls, to regular flooring, to your clothing or even the fabric of that clothing—would be possible without the precise measurement that is itself reliant on numerical, rather than quantical, cognition.

One major socio-cultural shift that is at least partially contingent on numerical cognition is urbanization. Urbanization, of course, was a byproduct of dense settlement patterns that were made possible by agriculture, since the latter allowed for the food stores requisite of densely structured populations. In contrast, hunting, gathering, and horticulture require less densely structured groups, and cannot sustain large groups of people in packed configurations. Agriculture, in turn, relies heavily on numerical cognition in ways that hunting and gathering do not. Inter alia, much of agriculture relies on the precise discrimination of astronomical patterns, and on the precise measurement of seeds, tilled rows, and so forth. Mathematics evolved only

after the advent of agriculture that enabled urban settlements with diverse vocations, including some that did not contribute directly to food production. These and other factors suggest that agriculture and mathematics coevolved (like much of human culture), benefiting each other in direct and indirect ways. One of many critical results of this coevolution was the advent of literacy, which is arguably a byproduct of symbolic notations developed in Mesopotamia and elsewhere to track quantities of grain and other agricultural products.

The pervasive cognitive, material, and socio-cultural effects of truly numerical thought may obscure the fact that these ubiquitous effects are culturally and linguistically contingent and certainly not native characteristics of our species. It was a multi-stage process for our species to invent number words and develop systems of notion that suit our purposes and externalize information in a way that lightens our cognitive load. These linguistic and symbolic inventions, if you will, are the direct and indirect products of the outgrowth of numerical cognition from quantal cognition—an outgrowth made possible by the structure of our bodies and only then externalized in the material record.

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